Problem 1

a) Mean = 235

$\sigma = 35$

$n = 75$

a) $\mu_{\bar{x}} = \mu = 235$

b) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{75}} = 4.041$ 3d.p

c) $\mu > 233$

$$Z\text{-score} = \frac{X-\mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{233-235}{35/\sqrt{75}} = \frac{-2}{4.041} = -0.4948$$

P-value from tables = 0.31207.
= 0.312 (3 d.p)

(d) $\mu < 225$

$$Z\text{-score} = \frac{X-\mu}{\sigma/\sqrt{n}} = \frac{225-235}{35/\sqrt{75}} = \frac{-10}{4.041}$$

Z-score = -2.474

Corresponding P-value = 0.0068
= 0.007 3dp.

e) $230 \leq \mu \leq 240$

$P(230 \leq \mu \leq 240) = P(\mu=240) - P(\mu=230)$.

Z-score for 230

$$Z = \frac{X-\mu}{\sigma/\sqrt{n}} = \frac{230-235}{35/\sqrt{n}} = -1.237.$$

$P(x \geq 230) = 0.1080$

Z-score for 240 $= \frac{240-235}{35/\sqrt{75}} = 1.237.$

P-value for $\mu = 240 = 0.1080$

Overall $P(230 \leq x \leq 240) = 0.7839$
$= 0.784$ 3 dp.

Problem 2

a) $\sigma = 5.8$ min's
$n = 38$
$\bar{x} = 40.7$
~~$\bar{x} = 40.7$~~

$\mu = \bar{x} - Z_i \sigma_{\bar{x}}$ $\Rightarrow \sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{5.8}{\sqrt{38}} = 0.941$
$\mu = 40.7 - Z(0.941)$

With 95% confidence interval
With a two-tailed test
Z-score $= 1.96$

$\therefore \mu = 40.7 - 1.96(0.941) = 38.856$

Confidence interval
$\bar{x} \pm Z_{\alpha/2} \cdot \sigma/\sqrt{n}$

$= 40.7 \pm 1.96(0.941) = 38.856, 42.544$

b) At an alpha level of 5% two tailed test, there is a 95% probability that the mean lies between 38.856 and 42.544

Problem 3

$n = 2400$

1680 believe Covid-19 is the biggest pandemic

a) $n = 2400$ (Sample size)

b) $p = \frac{1680}{2400} = 0.7$

c) $q = (1-p) = (1-0.7) = 0.3$

d) 98% confidence interval means $\alpha = 2\%$

$\alpha/2 = 1\%$

Z-score from tables $= 2.32$ or 2.33

e) $p = 0.7$

Confidence interval, Taking $\sigma_{\bar{x}} = 1$

$\bar{p} \pm Z_{score}$

$0.7 \pm 0.01 = 0.69, 0.71$

f) The proportion of the population that believes the country's biggest problem is COVID-19 is between 0.69 and 0.71 when the confidence level is 0.98 or 98%

Problem 4

$p = 0.5$

$q = 0.5$

92% confidence $\alpha = 8$ $\alpha/2 = 4\%$, Z-score $= 1.75$

Estimated proportion is within [illegible] 0.05

[illegible]

$EBP = 0.05$

$Z_{0.04} = 1.75$

$p'q' = 0.5^2 = 0.25$

$n = \frac{Z^2 p'q'}{EBP^2} = \frac{1.75^2 \times 0.25}{0.05^2} = 306.25$

$= \underline{307}$

Problem 5

$\sigma = 0.25$

$\mu = 1.10$

$n = 52$

$\bar{x} = 0.98$

$\sigma = 0.15$

$\alpha = 0.01$

Hypothesis

$H_0 : \mu = 1.10$

$H_1 : \mu \neq 35$

$$Z\text{-statistic} = \frac{\bar{x} - \mu_0}{\sigma_0 / \sqrt{n}} = \frac{0.98 - 1.10}{0.25 / \sqrt{52}} = -3.461$$

Critical values of the two tailed test is determined using the formula

$$Z_{\alpha/2} = Z_{0.005} = -2.57$$

Since $|z| > |Z_{crit}|$ there is no enough evidence to conclude that $\mu \neq 1.10$

Problem 6

$p = 0.75$

$q = 0.25$

$n = 100$

$\hat{p} = 0.78$

$\alpha = 0.03$

$H_0 : \mu \geq 0.75$

$H_1 : \mu \ngtr 0.75$

~~Z-statistic = 0.78 - 0.75~~

$$\hat{p} = \frac{x}{n} = \frac{78}{100} = 0.78$$

For a 2 tailed test using $\alpha = 0.03$

$$Z_{\frac{0.03}{2}} = Z_{0.015} = 0.50599$$

$$\sigma = \sqrt{npq}$$

$$\sigma = \sqrt{100 \times 0.78 \times 0.22} = 4.14$$

$$Z\text{-stat} = \frac{\bar{x} - \mu_0}{\sigma_0/\sqrt{n}} = \frac{0.78 - 0.75}{4.14/\sqrt{100}}$$

$$= \frac{0.03}{0.414} = 0.072$$

Since $|z| < |Z_{crit}|$ there is no enough evidence to deny that $H_0 : \mu = 0.75$